Domácí úkol – 1. ročník

Příklad 1:

$$\left(a\cdot\sqrt{\frac{b}{a}}+\frac{b}{1-\sqrt{\frac{b}{a}}}\right):\frac{b+\sqrt{ab}}{b\left(\frac{1}{b}-\frac{1}{a}\right)}=$$

Upravíme první závorku:

$$a\cdot\sqrt{\frac{b}{a}}+\frac{b}{1-\sqrt{\frac{b}{a}}}=\frac{a\sqrt{\frac{b}{a}}\cdot\left(1-\sqrt{\frac{b}{a}}\right)+b}{1-\sqrt{\frac{b}{a}}}=$$

$$=\frac{a\sqrt{\frac{b}{a}}-\cancel{a}\frac{b}{\cancel{a}}+b}{1-\sqrt{\frac{b}{a}}}=\frac{a\sqrt{\frac{b}{a}}}{1-\sqrt{\frac{b}{a}}}=\frac{\sqrt{ab}}{1-\sqrt{\frac{b}{a}}}=\frac{a\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

Upravíme druhý zlomek:

$$\frac{b+\sqrt{ab}}{1-\frac{b}{a}}=\frac{ab+a\sqrt{ab}}{a-b}$$

Obě úpravy spojíme podle zadání:

$$\frac{a\sqrt{b}}{\sqrt{a}-\sqrt{b}}\cdot\frac{a-b}{ab+a\sqrt{ab}}=\frac{a\sqrt{b}}{\cancel{\sqrt{a}-\sqrt{b}}}\cdot\frac{\overbrace{(a-b)}^{(\sqrt{a}+\sqrt{b})\cancel{(\sqrt{a}-\sqrt{b})}}}{a(b+\sqrt{ab})}=$$

$$=\frac{a\sqrt{ab}+ab}{ab+a\sqrt{ab}}=\underline{\underline{1}}$$

$$[1\wedge ab>0\wedge a\neq b]$$

Příklad 2:

Nalezněte graf funkce:

$$f: y = \frac{\left(x^2 - 3x + 2\right) \cdot \left|x + 2\right|}{x^3 - x^2 - 4x + 4}$$

Upravíme trojčlen v čitateli:

$$x^2 - 3x + 2 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \begin{cases} x_1 = 2 \\ x_2 = 1 \end{cases}$$

$$x^2 - 3x + 2 = (x-2)(x-1)$$

Upravíme jmenovatele:

$$x^3 - x^2 - 4x + 4 = x^2(x-1) - 4(x-1) = (x-1)(x^2-4) =$$

$$= (x-1)(x-2)(x+2)$$

Upravíme celkový výraz:

$$f: y = \frac{\cancel{(x-2)}\cancel{(x-1)} \cdot |x+2|}{\cancel{(x-1)}\cancel{(x-2)}(x+2)} = \frac{|x+2|}{x+2}$$

$$NB: \; x + 2 = 0, \quad \underline{x = -2}$$

$$1) \; (-\infty; -2) \quad |x+2| = -(x+2)$$

$$y = -1$$

$$2) \; (-2; \infty) \quad |x+2| = x+2$$

$$y = 1$$

$$f: y = \begin{cases} -1 \; pro \quad x \in (-\infty; -2) \\ 1 \; \; pro \, x \in (-2; \infty) - \{1; 2\} \end{cases}$$

Příklad 3:

$$\frac{a\cdot\left(\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1}+b\cdot\left(\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}}{\left(\frac{a+\sqrt{ab}}{2ab}\right)^{-1}+\left(\frac{b+\sqrt{ab}}{2ab}\right)^{-1}}=$$

$$a\cdot\frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}+\frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}\cdot b=\frac{2ab\cancel{(\sqrt{a}+\sqrt{b})}}{\cancel{\sqrt{a}+\sqrt{b}}}=2ab$$

$$\frac{2ab}{a+\sqrt{ab}}+\frac{2ab}{b+\sqrt{ab}}=\frac{2ab(b+\sqrt{ab}+a+\sqrt{ab})}{(a+\sqrt{ab})(b+\sqrt{ab})}$$

$$=\frac{\cancel{2ab}\cdot(a+\sqrt{ab})(b+\sqrt{ab})}{\cancel{2ab}(a+2\sqrt{ab}+b)}=\frac{ab+b\sqrt{ab}+a\sqrt{ab}+ab}{a+2\sqrt{ab}+b}=$$

$$=\frac{a\sqrt{ab}+b\sqrt{ab}+2ab}{a+2\sqrt{ab}+b}=\frac{\sqrt{ab}\cancel{(a+b+2\sqrt{ab})}}{\cancel{a+2\sqrt{ab}+b}}=\sqrt{ab}$$

$\sqrt{ab}\wedge a>0\wedge b>0$

Příklad 4:

$$\frac{\frac{a^4-b^4}{a^2b^2}}{\left(1+\frac{b^2}{a^2}\right)\cdot\left(1-\frac{2a}{b}+\frac{a^2}{b^2}\right)}=$$

$$a^4-b^4=(a^2-b^2)(a^2+b^2)=(a-b)(a+b)(a^2+b^2)$$

$$1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}\qquad 1-\frac{2a}{b}+\frac{a^2}{b^2}=\frac{b^2-2ab+a^2}{b^2}=\frac{(b-a)^2}{b^2}$$

$$\frac{\frac{(a-b)(a+b)(a^2+b^2)}{a^2b^2}}{\frac{a^2+b^2}{a^2}\cdot\frac{(b-a)^2}{b^2}} = \frac{a^2b^2(a-b)(a+b)\cancel{(a^2+b^2)}}{\cancel{(a^2+b^2)}\underbrace{(b-a)^2}_{(a-b)^2}a^2b^2} =$$

$$= \frac{\cancel{a^2b^2}(a+b)}{(a-b)\cancel{(a^2b^2)}} = \frac{a+b}{a-b}$$

$$\frac{a+b}{a-b} \wedge a \neq b \wedge a \neq 0 \wedge b \neq 0$$

Příklad 5:

$$6x+\left(\frac{x}{x-2}-\frac{x}{x+2}\right):\frac{4x}{x^4-2x^3+8x-16}=$$

$$6x+\left(\frac{x}{x-2}-\frac{x}{x+2}\right):\frac{4x}{\underbrace{x^4-2x^3}_{x^3(x-2)}+\underbrace{8x-16}_{8(x-2)}}=$$

$$=6x+\frac{\cancel{4x}}{\cancel{(x-2)}(x+2)}\cdot\frac{\cancel{(x-2)}(x^3+8)}{\cancel{4x}}=$$

$$x^3+8=\cancel{(x+2)}(x^2-2x+4)$$

$$6x+\frac{x^3+8}{\cancel{x+2}}=6x+x^2-2x+4=x^2+4x+4=(x+2)^2$$

$$(x+2)^2 \wedge x \neq 0 \wedge x \neq \pm 2$$

Příklad 6:

$$\frac{x^2+xy+y^2}{x}:\left(\frac{x^2+y^2}{x^2y^2}\cdot\frac{x^3-y^3}{x^2+y^2}\right)=$$

$$\frac{x^2+xy+y^2}{x}:\left(\frac{x^2+y^2}{x^2y^2}\cdot\frac{x^3-y^3}{x^2+y^2}\right)=\frac{x^2+xy+y^2}{x}\cdot\frac{x^2y^2}{(x-y)(x^2+xy+y^2)}=\frac{xy^2}{x-y};$$

Podm.: $x\neq 0, y\neq 0, x\neq y$

Příklad 7:

$$6a+\left(\frac{a}{a-2}-\frac{a}{a+2}\right):\frac{4a}{a^4-2a^3+8a-16}=$$

$$6a+\left(\frac{a}{a-2}-\frac{a}{a+2}\right):\frac{4a}{a^4-2a^3+8a-16}=6a+\frac{a^2+2a-a^2+2a}{(a-2)(a+2)}\cdot\frac{a^3(a-2)+8(a-2)}{4a}=$$

$$6a+\frac{4a(a-2)(a^3+8)}{4a(a-2)(a+2)}=6a+\frac{(a+2)(a^2-2a+4)}{a+2}=a^2+4a+4=(a+2)^2;$$

Podm.: $a\neq 0, a\neq \pm 2$

Příklad 8:

$$\left[\frac{a^3-ab^2+b^3}{(a-b)^3}-\frac{b}{a-b}\right]\left[\frac{a^2-2ab+2b^2}{a^2-ab+b^2}-\frac{b}{a}\right]=$$

$$\left[\frac{a^3-ab^2+b^3}{(a-b)^3}-\frac{b}{a-b}\right]\left[\frac{a^2-2ab+2b^2}{a^2-ab+b^2}-\frac{b}{a}\right]=$$

$$\frac{a^3-ab^2+b^3-b(a^2-2ab+b^2)}{(a-b)^3}\cdot\frac{a^3-2a^2b+2ab^2-a^2b+ab^2-b^3}{(a^2-ab+b^2).a}=$$

$$=\frac{a^3-ab^2+b^3-a^2b+2ab^2-b^3}{(a-b)^3}\cdot\frac{a^3-3a^2b+3ab^2-b^3}{(a^2-ab+b^2).a}=$$

$$=\frac{\cancel{a^3+ab^2-a^2b}}{(a-b)^3}\cdot\frac{(a-b)^3}{\cancel{a^3+ab^2-a^2b}}=1;$$

Podm.: $a\neq 0, a\neq b$

Příklad 9:

$$\left[y^2 - \frac{x}{1-\frac{x}{y-x}} \cdot \left(\frac{xy}{y-x} - x\right)\right] : \frac{x^2+xy+y^2}{y} =$$

$$\left[y^2 - \frac{x}{1+\frac{x}{y-x}} \cdot \left(\frac{xy}{y-x} - x\right)\right] : \frac{x^2+xy+y^2}{y} =$$

$$= \left[y^2 - \frac{x(y-x)}{y} \cdot \frac{x^2}{y-x}\right] \cdot \frac{y}{x^2+xy+y^2} = \frac{y^3-x^3}{y} \cdot \frac{y}{x^2+xy+y^2} =$$

$$= \frac{(y-x)(x^2+xy+y^2)}{x^2+xy+y^2} = y-x;$$

Příklad 10:

Zjednodušte výraz: $\frac{x^2 - 9xy + 14y^2}{x^2 - xy - 2y^2} =$

$$\frac{x^2-9xy+14y^2}{x^2-xy-2y^2} = \frac{(x-7y)(x-2y)}{(x-2y)(x+y)} = \frac{x-7y}{x+y}.$$

Podm.: $x \neq -y,\ x \neq 2y$

Příklad 11:

Řešte v R:

$(x^2 + 2x - 3)(x^2 + 2x + 1) - 5 = 0$

$$(x^2 + 2x - 3)(x^2 + 2x + 1) - 5 = 0$$

Subst.: $x^2 + 2x - 3 = a$

$x^2 + 2x + 1 = a + 4$

$$a(a + 4) - 5 = 0$$
$$a^2 + 4a - 5 = 0$$
$$(a + 5)(a - 1) = 0$$
$$a = -5 \vee a = 1$$

1. $x^2 + 2x - 3 = -5$

   $x^2 + 2x + 2 = 0$

   $D = 4 - 8 = -4 < 0$

   $K_1 = \emptyset$

2. $x^2 + 2x - 3 = 1$

   $x^2 + 2x - 4 = 0$

   $x_{1,2} = \frac{-2\pm\sqrt{4+16}}{2} = -1 \pm \sqrt{5}$

   $K_2 = \{-1 \pm \sqrt{5}\}$

$$K = K_1 \cup K_2 = \{-1 \pm \sqrt{5}\};$$

Příklad 12:

Řešte v R:

$$\frac{1}{3}\left(\frac{2x-1}{x^2}+2\right)\left(\frac{1-2x}{x^2}\right)=\left(3\cdot\frac{2x-1}{x^2}\right)^2$$

Podm.: $x \neq 0$

$$\frac{1}{3}\left(\frac{2x-1}{x^2}+2\right)\left(\frac{1-2x}{x^2}+1\right)=\left(3.\frac{2x-1}{x^2}\right)^2$$

$$\frac{1}{3}(a+2)(-a+1)=(3a)^2/.3$$

$$-a^2-a+2=27a^2$$

$$28a^2+a-2=0$$

Subst.: $\frac{2x-1}{x^2}=a,$ $\frac{1-2x}{x^2}=-a$

$$a_{1,2}=\frac{-1\pm\sqrt{1+224}}{56}=\frac{-1\pm15}{56}=<\begin{matrix}\frac{1}{4}\\ -\frac{2}{7}\end{matrix}$$

1. $\frac{2x-1}{x^2}=\frac{1}{4}$

$$x^2-8x+4=0$$

$$x_{1,2}=\frac{8\pm\sqrt{64-16}}{2}=\frac{8\pm4\sqrt{3}}{2}=4\pm2\sqrt{3}$$

2. $\frac{2x-1}{x^2}=-\frac{2}{7}$

$$2x^2+14x-7=0$$

$$x_{1,2}=\frac{-14\pm\sqrt{196+56}}{4}=\frac{-14\pm6\sqrt{7}}{4}=\frac{-7\pm3\sqrt{7}}{2}$$

$$K=\left\{4\pm2\sqrt{3};\ \frac{-7\pm3\sqrt{7}}{2}\right\}$$